A numerical modelling of elastic fluid-structure interaction for aquaculture applications

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Concern

- Open water environment
  - Higher force and larger motion response
  - Non-linear wave-structure interaction

Photo by Alex Antoniadis

Figure by Fusion maritime Ltd.

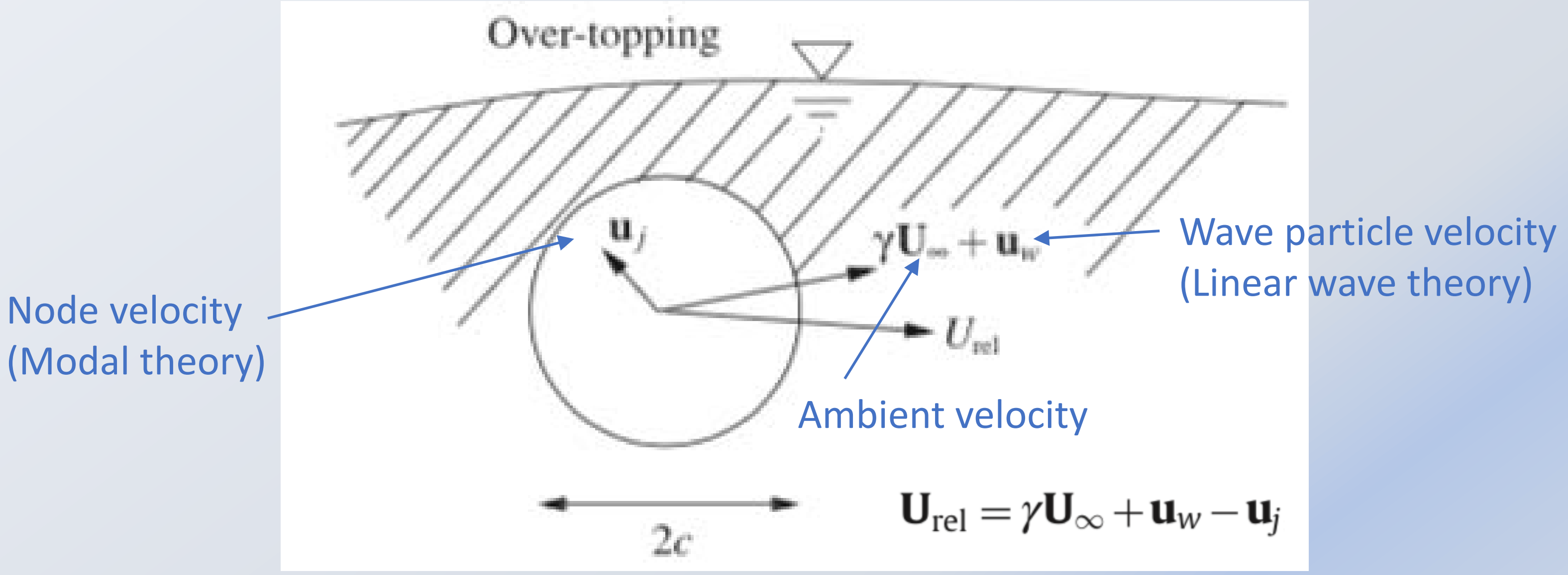
Structural damage

- Cooke aquaculture, Nova Scotia, Canada (2018)
  - Winter storm damage
  - Fish escapement

https://responsibleaquaculture.wordpress.com

State-of-the-art

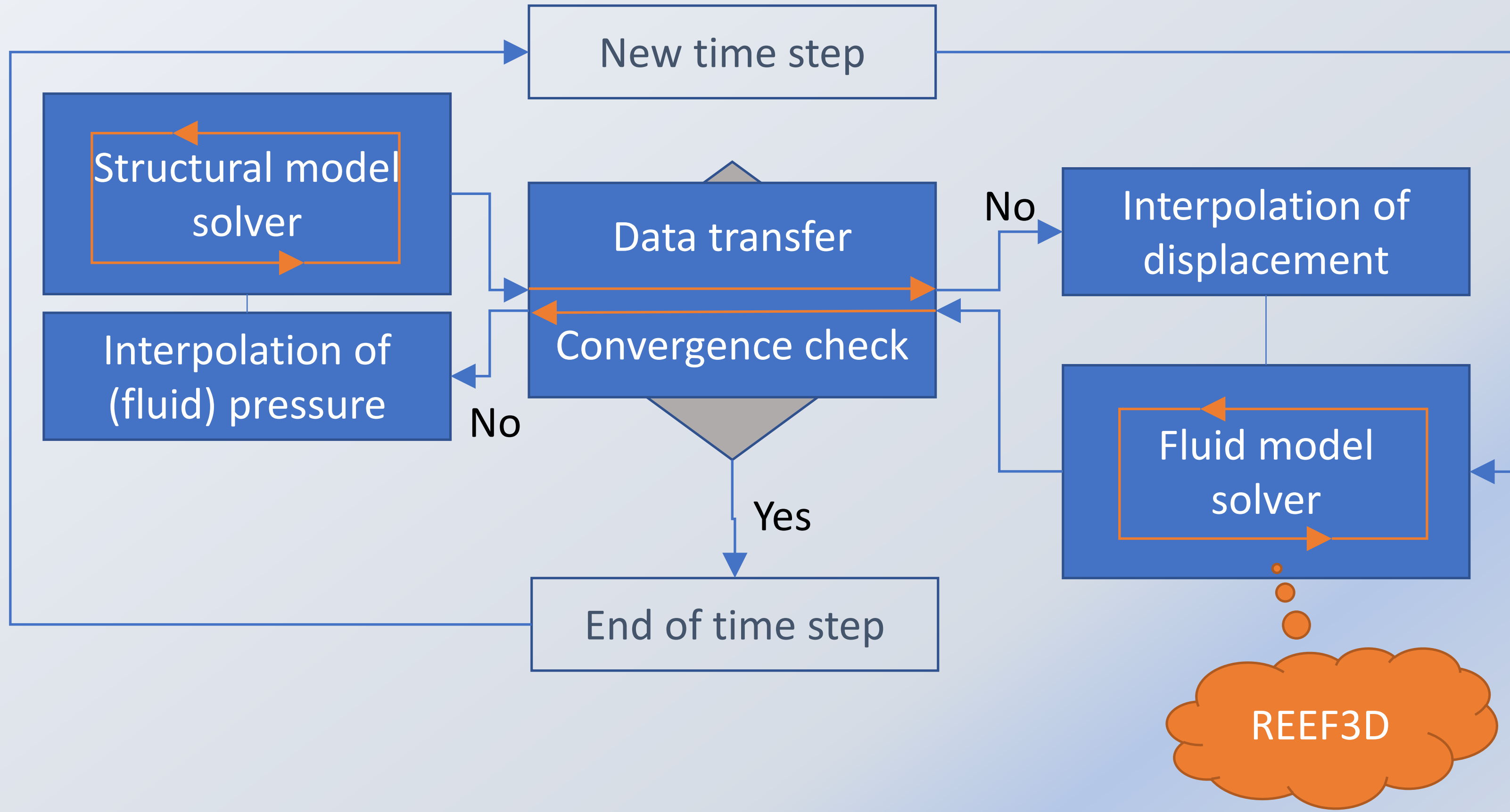
- Xu et al. (2017)
  - Physical model test and finite element (FE) analysis for collar only
- Kristiansen and Faltinsen (2015)
  - Rigid body equation with nonlinear hydrodynamic forcing
  - Significant load increase in steeper waves



Kristiansen and Faltinsen (2015)

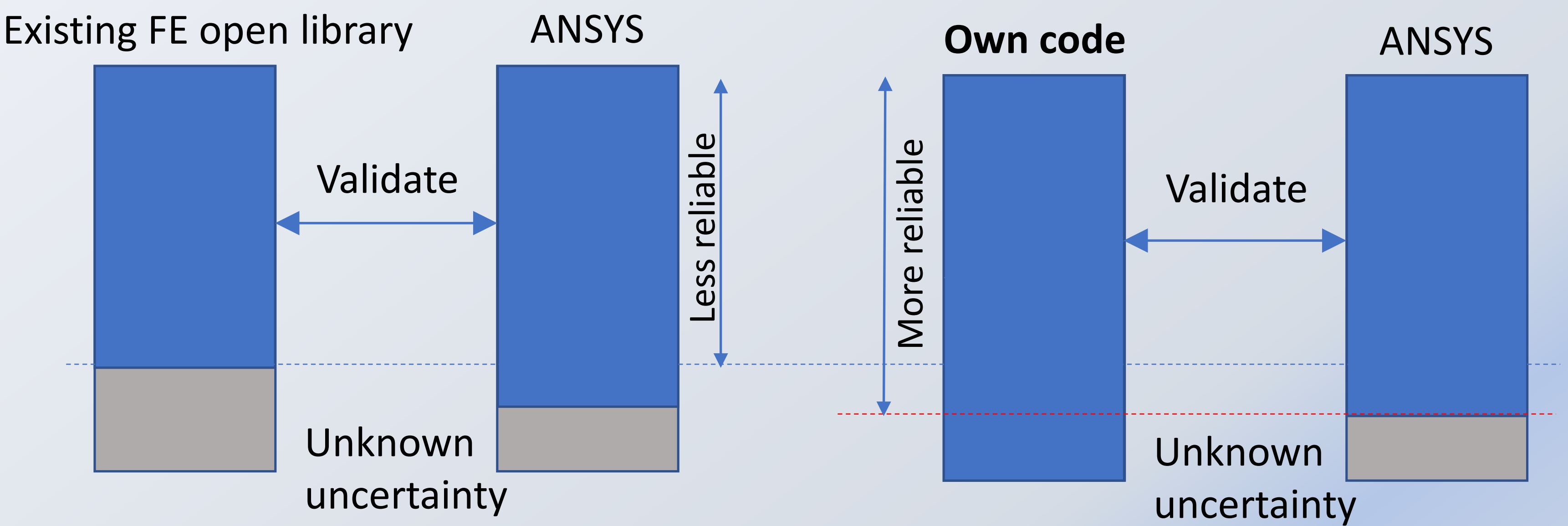
Work flow

- Computational Fluid Dynamics
- Fluid structure interaction



Why develop a new code?

- To extend higher order scheme
- Same amount of validating work
- Less unknown uncertainties



Governing equation – Structure model

$$[\boldsymbol{M}]\{\ddot{u}(t)\}+[\boldsymbol{K}]\{u(t)\} = \{F(t)\}$$

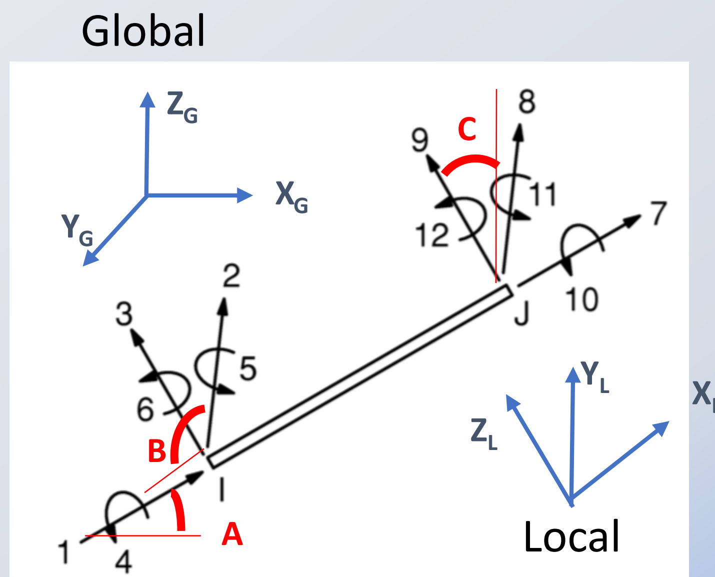
Where, u : Displacement vector, F : Force vector
$[\boldsymbol{M}]$: Mass matrix, $[\boldsymbol{K}]$: Stiffness matrix

Deflection

Torsion

Bending, ~~shear~~

$$\{u^e\} = \begin{Bmatrix} u_{xi} \\ u_{yi} \\ u_{zi} \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \\ u_{xj} \\ u_{yj} \\ u_{zj} \\ \theta_{xj} \\ \theta_{yj} \\ \theta_{zj} \end{Bmatrix}$$



$\{u^e\}$: 12 x 1 vector

Time integration - Newmark method

- Popular in fluid-Structure interaction
- 2nd order scheme

K. J. Bathe (1982)

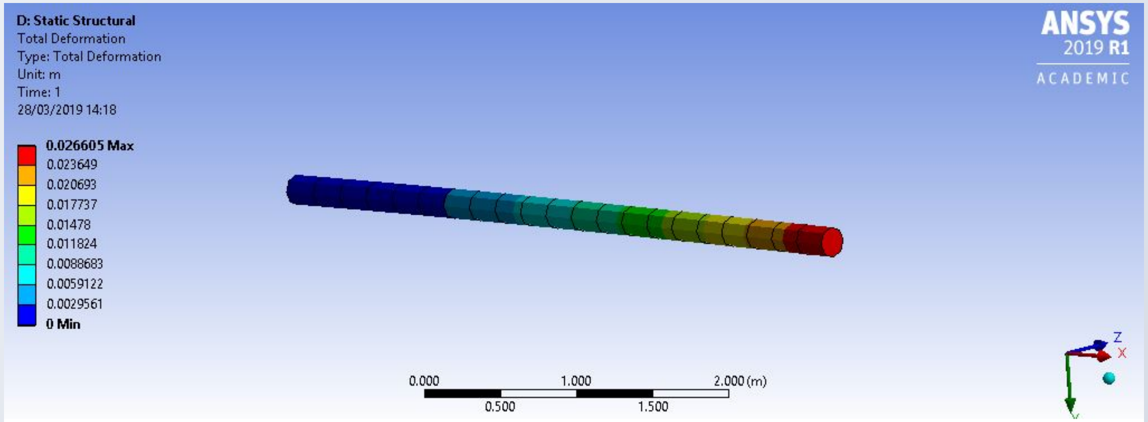
$$\dot{\boldsymbol{U}}_{\boldsymbol{t+\Delta t}} = \dot{\boldsymbol{U}}_{\boldsymbol{t}} + [\boldsymbol{\delta}\ddot{\boldsymbol{U}}_{\boldsymbol{t+\Delta t}} + (\mathbf{1} - \boldsymbol{\delta})\ddot{\boldsymbol{U}}_{\boldsymbol{t}}]\,\boldsymbol{\Delta t}$$

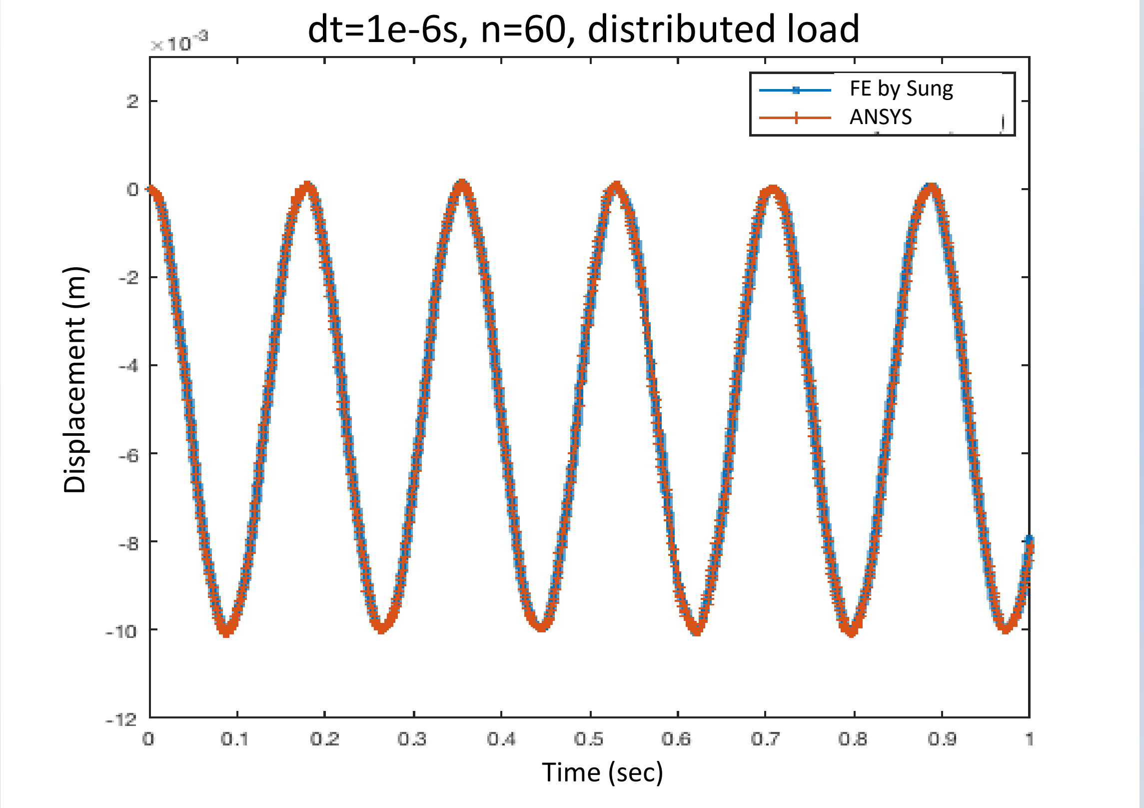
$$\boldsymbol{U}_{\boldsymbol{t+\Delta t}} = \boldsymbol{U}_{\boldsymbol{t}} + \dot{\boldsymbol{U}}_{\boldsymbol{t}}\boldsymbol{\Delta t} + [\boldsymbol{\alpha}\ddot{\boldsymbol{U}}_{\boldsymbol{t+\Delta t}} + (\frac{\mathbf{1}}{\mathbf{2}} - \boldsymbol{\alpha})\ddot{\boldsymbol{U}}_{\boldsymbol{t}}]\,\boldsymbol{\Delta t^2}$$

Where, $\boldsymbol{\delta} = \frac{\mathbf{1}}{\mathbf{2}}$, $\boldsymbol{\alpha} = \frac{\mathbf{1}}{\mathbf{4}}$

Validation Case – Steel cantilever

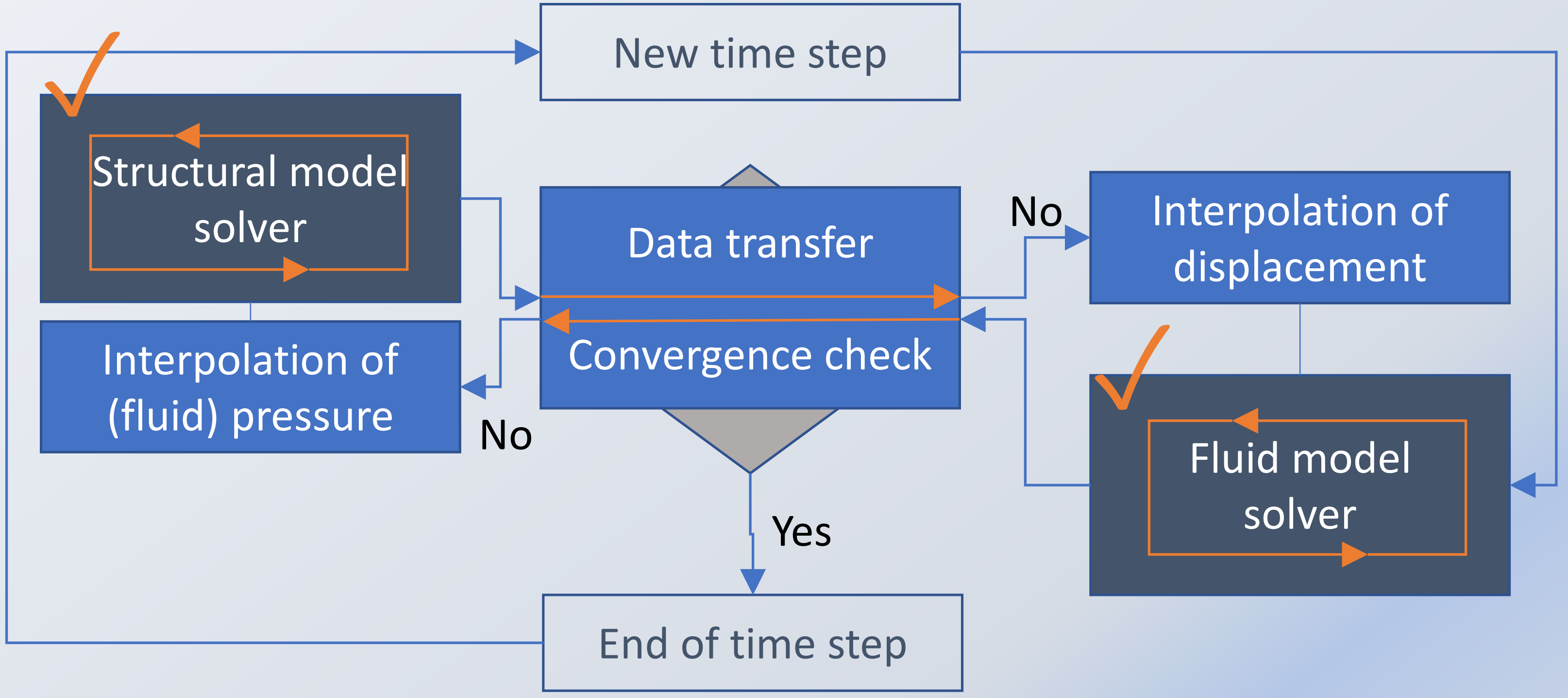
- Own FE code vs. ANSYS
- 2D, 3D Beam
- Static, Transient analysis
- Point, distributed load





Next step - Coupling

- Convergence check in displacement
- Data Transfer
- Interpolation



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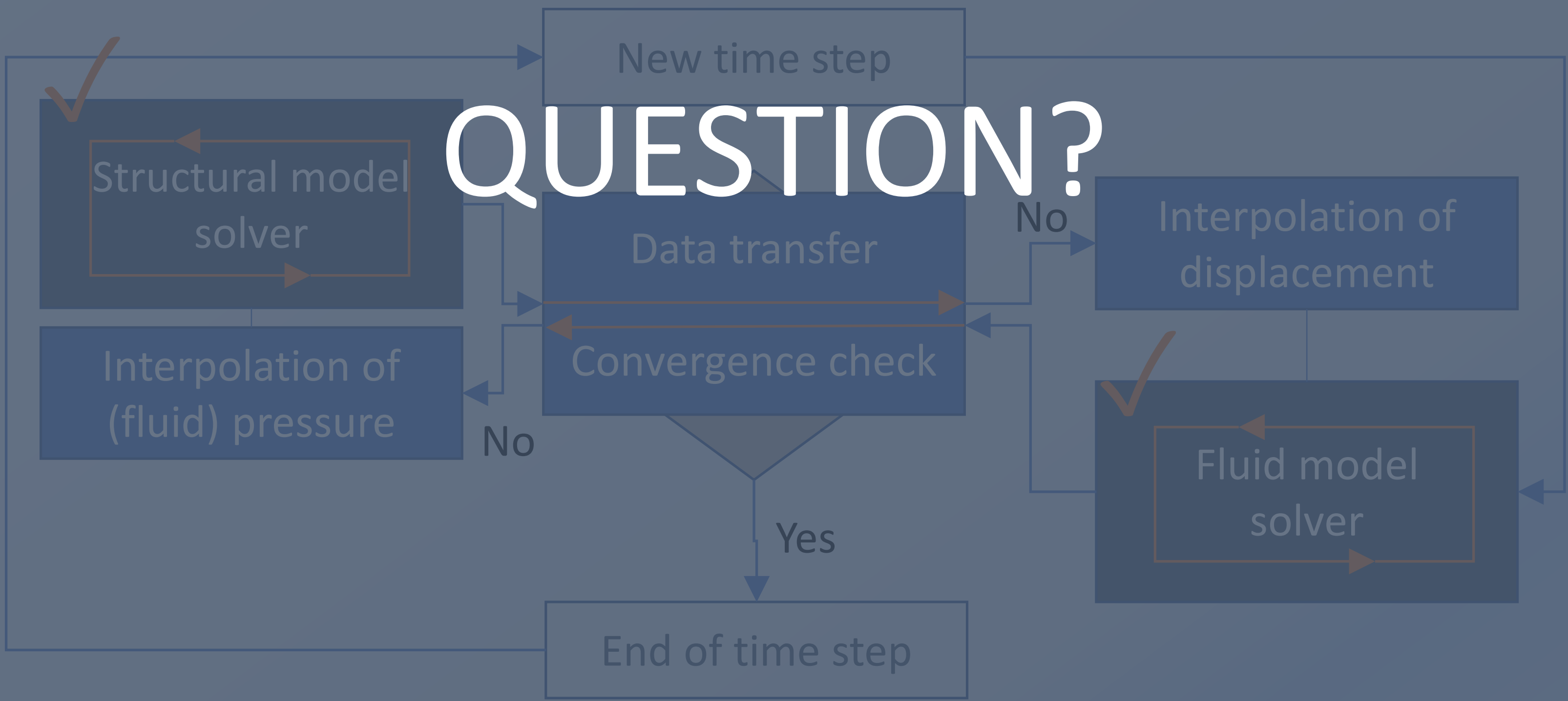
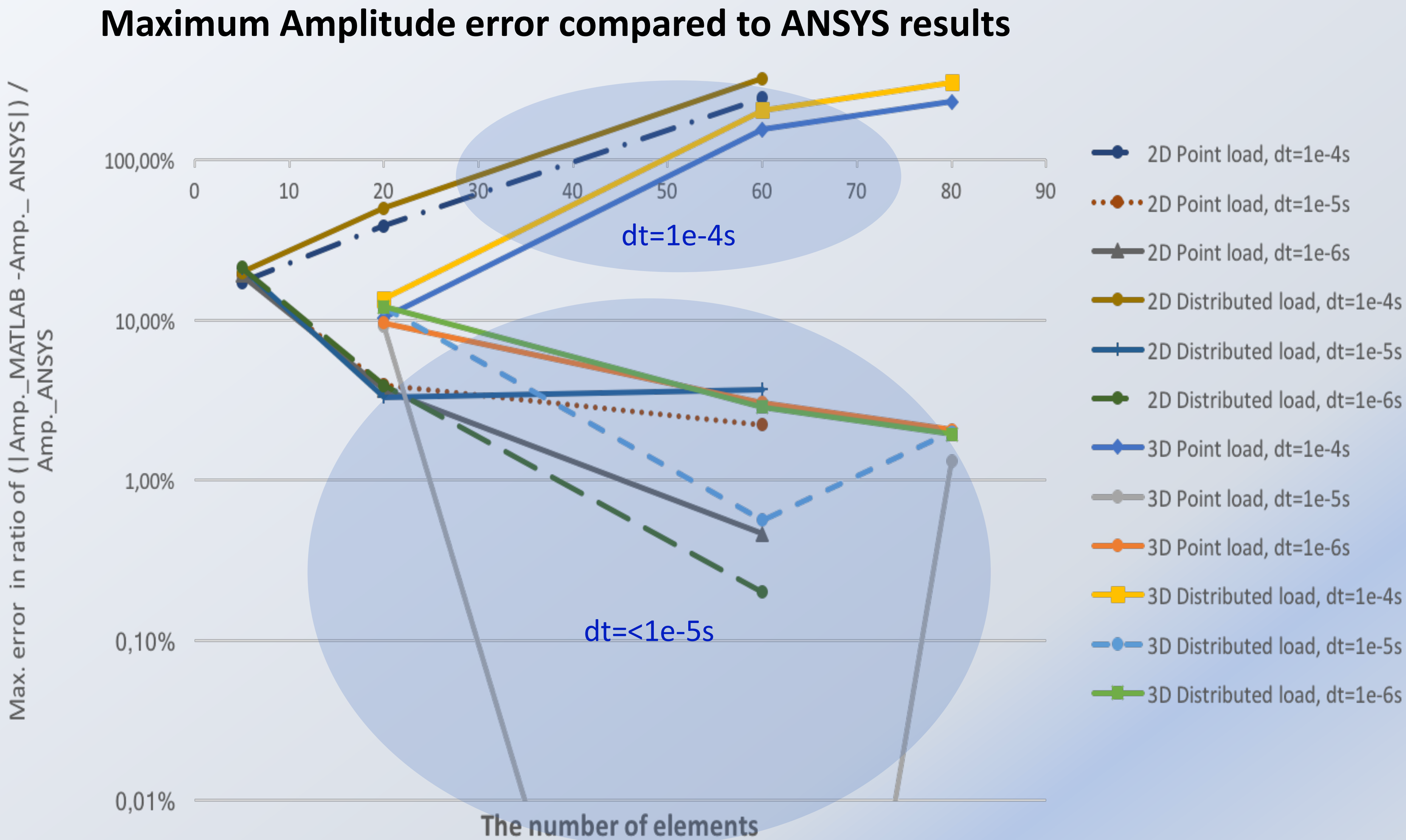
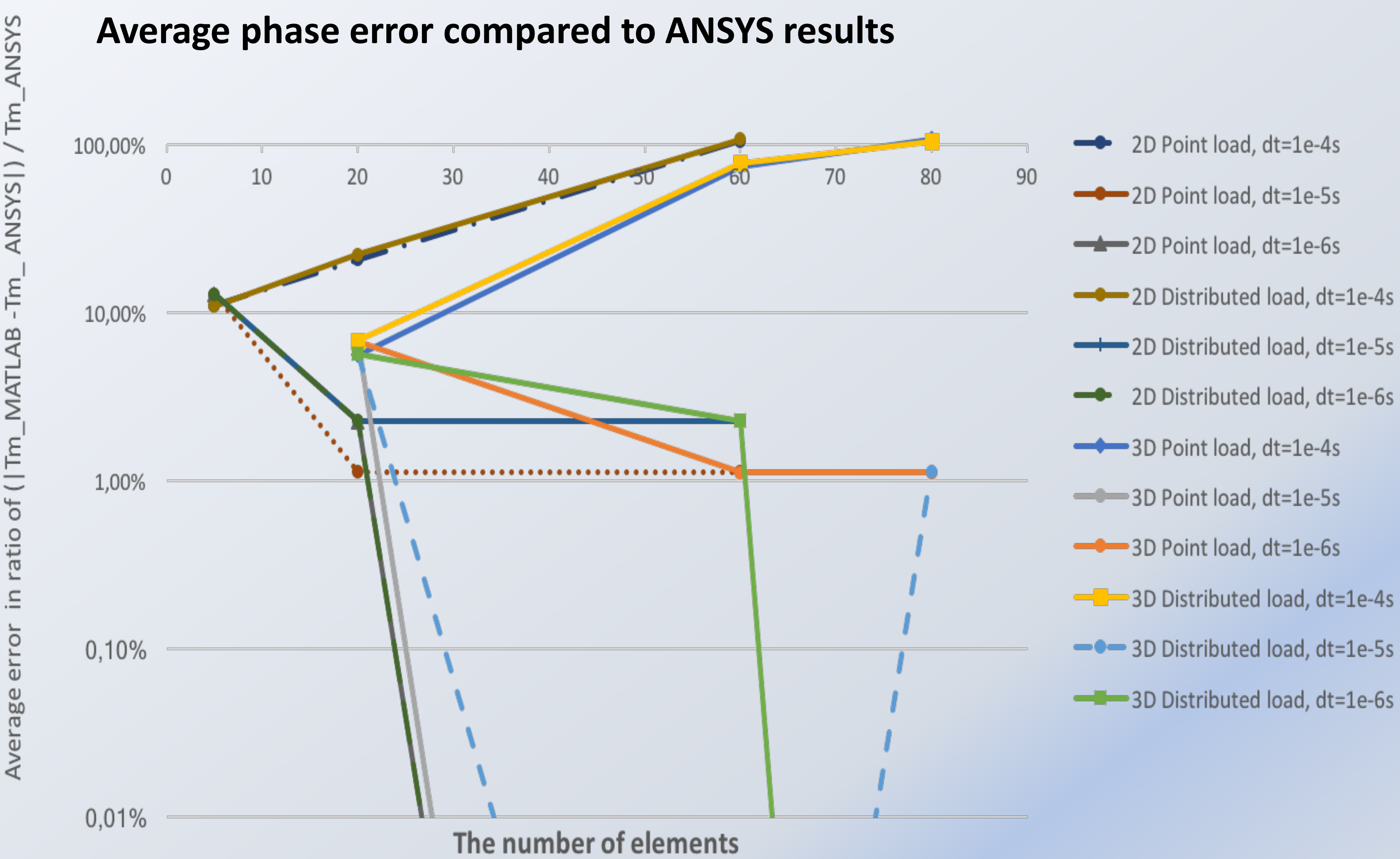


Back-up slide : Validation (1/2) – Amplitude error

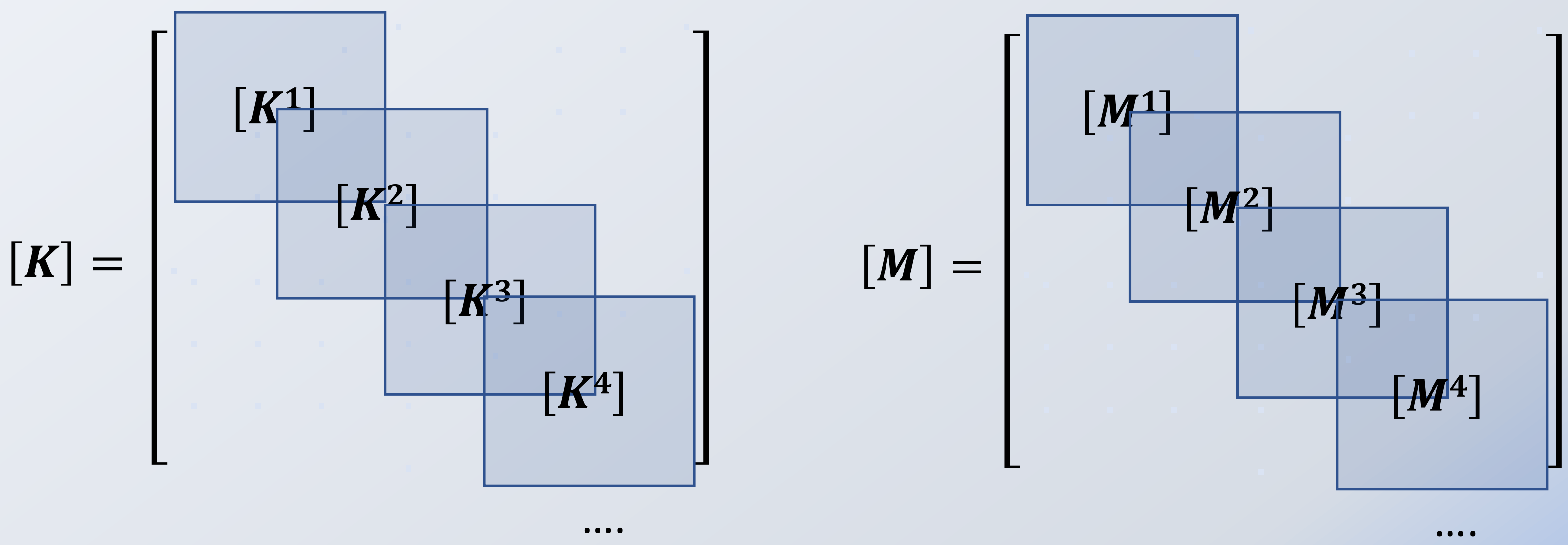


Back-up slide : Validation (2/2) – Phase error



Back-up slide : Governing equation – Structure model

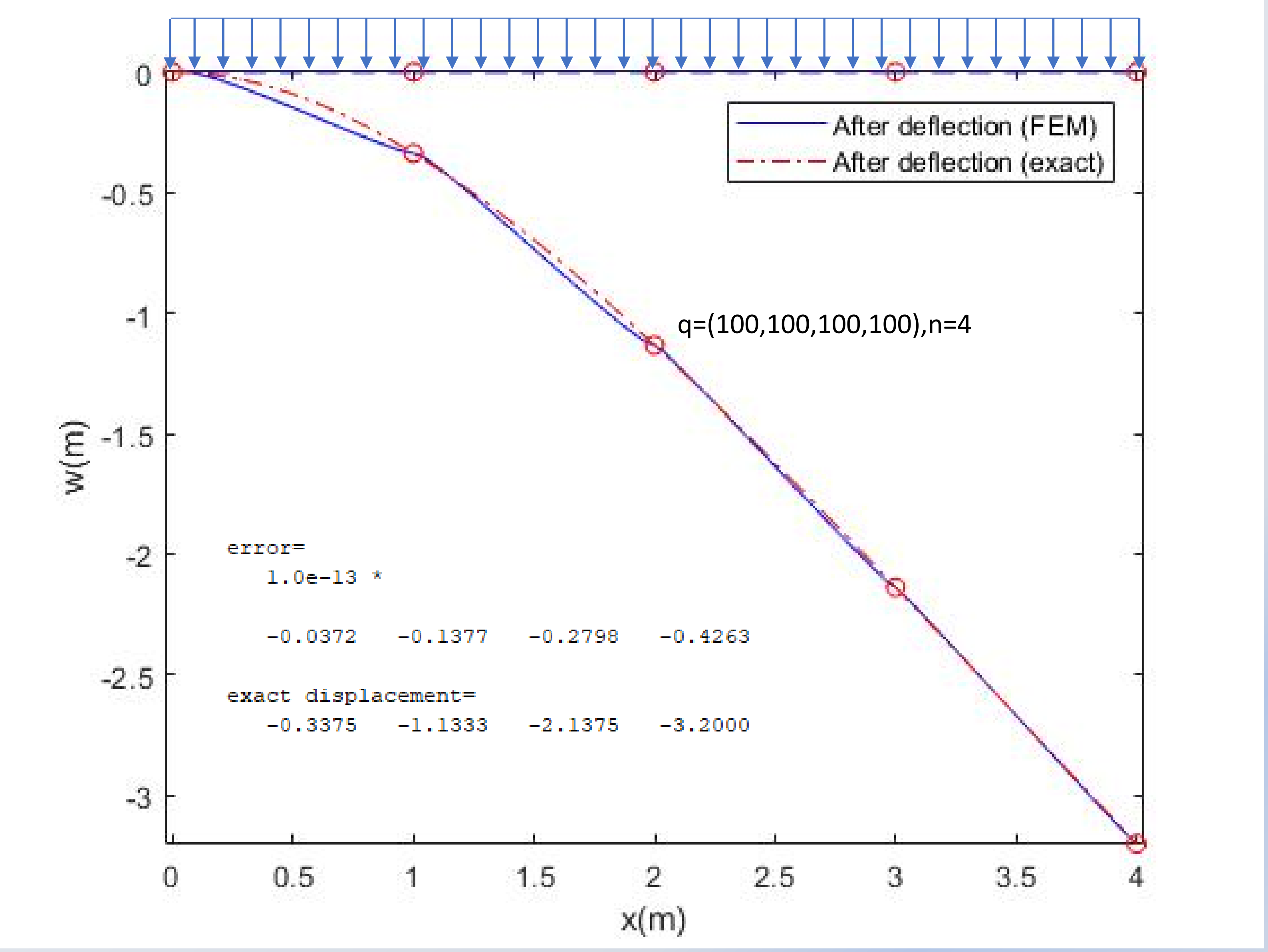
$$[\boldsymbol{M}]\{\ddot{u}(t)\}+[\boldsymbol{K}]\{u(t)\} = \{F(t)\}$$



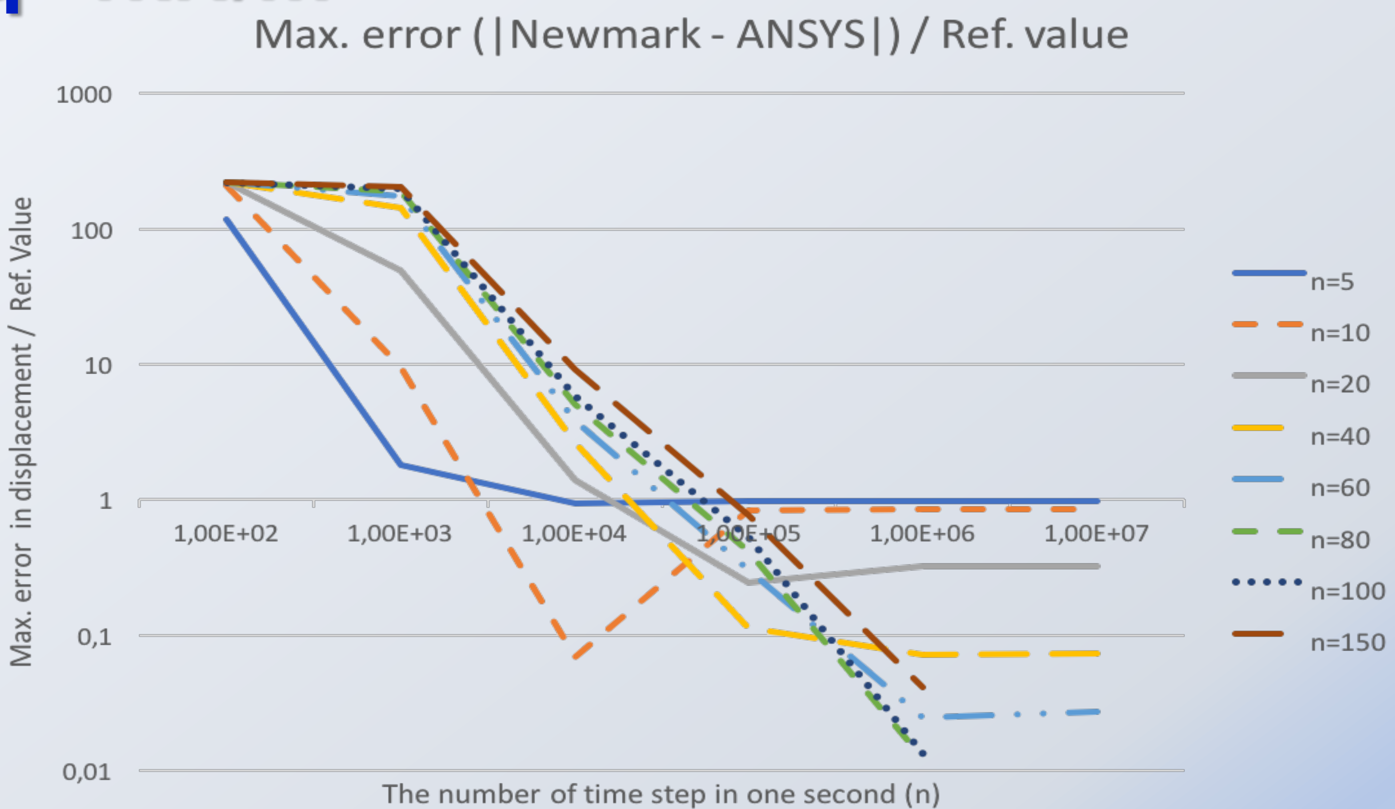
$[\boldsymbol{K}^e], [\boldsymbol{M}^e]$: 12 x 12 matrix, $[\boldsymbol{K}], [\boldsymbol{M}]$: N x N matrix

N : the degree of freedom = (# of element+1)x1 e.g. 9 elements, 120 dofs

Back up slide : Validation – Static, Distributed load



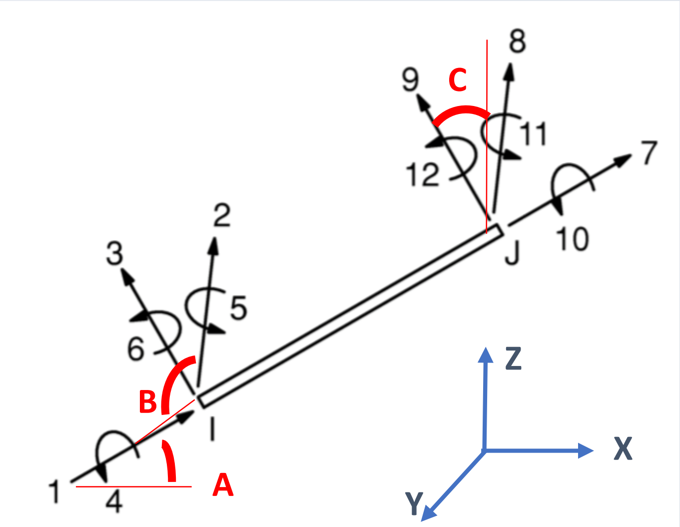
Back up slide: Convergence, 2D Transient, q=1kN/m



Max. Error in %
(Ref. displacement = 5.3E-03 (m))

# of timestep	1,00E+02	1,00E+03	1,00E+04	1,00E+05	1,00E+06	1,00E+07
Nele \ dt	1,00E-02	1,00E-03	1,00E-04	1,00E-05	1,00E-06	1,00E-07
5	11630 %	181 %	94 %	98 %	98 %	98 %
10	20858 %	955 %	7 %	83 %	85 %	85 %
20	22121 %	4938 %	140 %	25 %	32 %	32 %
40	22200 %	14234 %	264 %	11 %	7 %	7 %
60	22155 %	17462 %	383 %	30 %	2 %	3 %
80	22128 %	18866 %	506 %	42 %	1 %	
100	22057 %	19636 %	589 %	53 %	1 %	
150	21992 %	20574 %	915 %	75 %	2 %	

3-D Coordination, Conversion local to global



$$R_Z R_Y R_X$$

$$= \begin{pmatrix} \cos C & -\sin C & 0 \\ \sin C & \cos C & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos B & 0 & \sin B \\ 0 & 1 & 0 \\ -\sin B & 0 & \cos B \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos A & -\sin A \\ 0 & \sin A & \cos A \end{pmatrix}$$

$$= \begin{pmatrix} \cos C \cos B & -\sin C & \cos C \sin B \\ \sin C \cos B & \cos C & \sin C \sin B \\ -\sin B & 0 & \cos B \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos A & -\sin A \\ 0 & \sin A & \cos A \end{pmatrix}$$

$$= \begin{pmatrix} \cos C \cos B & -\sin C \cos A + \cos C \sin B \sin A & \sin C \sin A + \cos C \sin B \cos A \\ \sin C \cos B & \cos C \cos A + \sin C \sin B \sin A & -\cos C \sin A + \sin C \sin B \cos A \\ -\sin B & \cos B \sin A & \cos B \cos A \end{pmatrix}$$

3-D *"M"* Matrix for an element

$$
[M_\ell] = M_t \begin{bmatrix}
1/3 & & & & & & & & & & & \\
0 & A_z & & & & & & & & & & \\
0 & 0 & A_y & & & & & & & \text{Symmetric} & & \\
0 & 0 & 0 & J_x/3A & & & & & & & & \\
0 & 0 & -C_y & 0 & E_y & & & & & & & \\
0 & C_z & 0 & 0 & 0 & E_z & & & & & & \\
1/6 & 0 & 0 & 0 & 0 & 0 & 1/3 & & & & & \\
0 & B_z & 0 & 0 & 0 & D_z & 0 & A_z & & & & \\
0 & 0 & B_y & 0 & -D_y & 0 & 0 & 0 & A_y & & & \\
0 & 0 & 0 & J_x/6A & 0 & 0 & 0 & 0 & 0 & J_x/3A & & \\
0 & 0 & D_y & 0 & F_y & 0 & 0 & 0 & C_y & 0 & E_y & \\
0 & -D_z & 0 & 0 & 0 & F_z & 0 & -C_z & 0 & 0 & 0 & E_z
\end{bmatrix}
$$

$M_t = (\varrho A + \cancelto{0}{m})L(1-\cancelto{0}{\varepsilon^{in}})$

ϱ = density (input as DENS on **MP** command)

m = added mass per unit length (input as ADDMAS on **RMORE** command)

ε^{in} = prestrain (input as ISTRN on **RMORE** command)

$A_z = A(r_z, \phi_y)$

$A_y = A(r_y, \phi_z)$

$B_z = B(r_z, \phi_y)$

⋮

$F_z = F(r_z, \phi_y)$

$F_y = F(r_y, \phi_z)$

$$A(r,\phi) = \frac{\frac{13}{35} + \frac{7}{10}\cancelto{0}{\phi} + \frac{1}{3}\cancelto{0}{\phi^2} + \frac{6}{5}(r/L)^2}{(1+\cancelto{0}{\phi})^2}$$

$$B(r,\phi) = \frac{\frac{9}{70} + \frac{3}{10}\cancelto{0}{\phi} + \frac{1}{6}\cancelto{0}{\phi^2} - \frac{6}{5}(r/L)^2}{(1+\cancelto{0}{\phi})^2}$$

$$C(r,\phi) = \frac{\left(\frac{11}{210} + \frac{11}{120}\cancelto{0}{\phi} + \frac{1}{24}\cancelto{0}{\phi^2} + \left(\frac{1}{10} - \frac{1}{2}\cancelto{0}{\phi}\right)(r/L)^2\right)L}{(1+\cancelto{0}{\phi})^2}$$

⋮

3-D "*K*" Matrix for an element

$$
[K_\ell] = \left[\begin{array}{cccccccccccc}
AE/L & & & & & & & & & & & \\
0 & a_z & & & & & & & \text{Symmetric} & & & \\
0 & 0 & a_y & & & & & & & & & \\
0 & 0 & 0 & GJ/L & & & & & & & & \\
0 & 0 & d_y & 0 & e_y & & & & & & & \\
0 & c_z & 0 & 0 & 0 & e_z & & & & & & \\
-AE/L & 0 & 0 & 0 & 0 & 0 & AE/L & & & & & \\
0 & b_z & 0 & 0 & 0 & d_z & 0 & a_z & & & & \\
0 & 0 & b_y & 0 & c_y & 0 & 0 & 0 & a_y & & & \\
0 & 0 & 0 & -GJ/L & 0 & 0 & 0 & 0 & 0 & GJ/L & & \\
0 & 0 & d_y & 0 & f_y & 0 & 0 & 0 & c_z & 0 & e_y & \\
0 & c_z & 0 & 0 & 0 & f_z & 0 & d_y & 0 & 0 & 0 & e_z
\end{array}\right]
$$

where: $a(l,\phi) = \dfrac{12EI}{L^3(1+\phi)}$ 0

$b(l,\phi) = \dfrac{-12EI}{L^3(1+\phi)}$ 0

$c(l,\phi) = \dfrac{6EI}{L^2(1+\phi)}$ 0

$d(l,\phi) = \dfrac{-6EI}{L^2(1+\phi)}$ 0

$e(l,\phi) = \dfrac{(4+\phi)EI}{L(1+\phi)}$ 0

$f(l,\phi) = \dfrac{(2-\phi)EI}{L(1+\phi)}$ 0 0

where: $\phi_y = \dfrac{12EI_z}{GA_z^sL^2}$

$\phi_z = \dfrac{12EI_y}{GA_y^sL^2}$

Since, $GAL^2 >> EI$